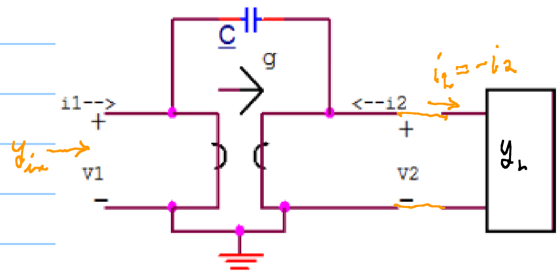
ENEE 610
09/15/20
corrected

Note Title 8/31/2020

Still base papers to choose
Today: loaded 2-port via Y's
Z, Vout/Vin and S matrices, passivity

Homework 2
Van der Pol ościllator



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} sC & -sC-g \\ -sC+g & sC \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix}$$

det Y Richards

$$= (sC)(sC) - (-sC-g)(-sC+g)$$
$$= (sC)^2 - (-sC)^2 + sCg - sCg + g^2 = g^2$$

$$-i_2 = i_L = y_L \cdot v_2$$

$\Rightarrow$ last row $(-sC+g)v_1 + sC\, v_2 = -y_L v_2$

$$(-sC - y_L) v_2 = (-sC+g) v_1$$

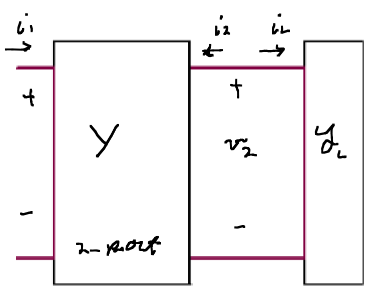
$$v_2 = \frac{-sC+g}{-sC-y_L} \cdot v_1$$

$$i_1 = sC v_1 + (-sC-g)\left[\frac{-sC+g}{-sC-y_L}\right] v_1$$

$$y_{in} = \frac{i_1}{v_1} = sC + \frac{(sC+g)(g-sC)}{sC+y_L}$$

$$= \frac{\cancel{(sC)^2} + sC y_L + \cancel{sCg} - \cancel{(sC)^2} + g^2 - \cancel{sCg}}{sC+y_L}$$

$$= \frac{sC y_L + g^2}{sC + y_L}.$$

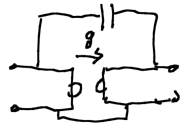


$$\therefore \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i_L = -i_2 = y_L v_2$$

$$i_2 = -y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow v_2 = \frac{-y_{21} v_1}{y_L + y_{22}}$$

$$i_1 = \left[y_{11} + \frac{(-y_{21}\, y_{12})}{y_L + y_{22}}\right] v_1$$

$$y_{in} = \frac{y_{11} y_L + y_{11} y_{22} - y_{12} y_{21}}{y_{22} + y_L} = \frac{y_{11} y_L + \det Y}{y_{22} + y_L}$$

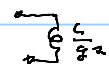


$Y = \begin{bmatrix} \alpha c & -\alpha c - g \\ -\alpha c + g & \alpha c \end{bmatrix} \Rightarrow i = Y v, \quad v = Z i; \quad Z = Y^{-1} = \frac{1}{\det Y}\begin{bmatrix} \alpha c & \alpha c + g \\ \alpha c - g & \alpha c \end{bmatrix} = \frac{1}{g^2}\begin{bmatrix} \alpha c & \alpha c + g \\ \alpha c - g & \alpha c \end{bmatrix}$

$z_{21} = \frac{\text{cofactor } (2,1) \text{ in } Y}{\det Y} = (-1)^{1+2} \cdot y_{12} = -y_{12}$

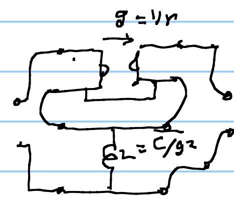
$v_1 = \alpha \frac{c}{g^2} i_1 + \left(\alpha \frac{c}{g^2} - \frac{1}{g}\right) i_2$

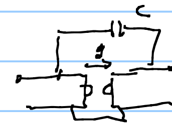
$v_2 = \alpha \frac{c}{g^2} i_1 + \frac{g}{g^2} i_1 + \alpha c i_2$



$\therefore \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \left\{ \begin{bmatrix} \alpha \frac{c}{g^2} & \alpha \frac{c}{g^2} \\ \alpha \frac{c}{g^2} & \alpha \frac{c}{g^2} \end{bmatrix} + \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \right\} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

$= \begin{bmatrix} \alpha L & \alpha L \\ \alpha L & \alpha L \end{bmatrix}, \quad L = c/g^2, \quad r = 1/g$





in VLSI $c = 10^{-9}$

$g = 10^{-3}$

$L = \frac{10^{-9}}{10^{-6}} = 10^{-3}$ = milliHy

$O_t = C i_b = [1_t \ K] i_b$

$= [O_t \ O_{t,l}] v_b$

$\begin{bmatrix} O_t & O_{t,l} \\ -K^T & 1_l \end{bmatrix} v_b = \begin{bmatrix} 1_t & K \\ O_{l,t} & O_l \end{bmatrix} i_b$ } connecting wires for the branch components

$\underbrace{\qquad}_{\text{KCL \& KVL}}$

$A v_b = B v_b$, i_a, v_a

$O_l = \mathcal{J} v_b = [-K^T \ 1_l] v_b$

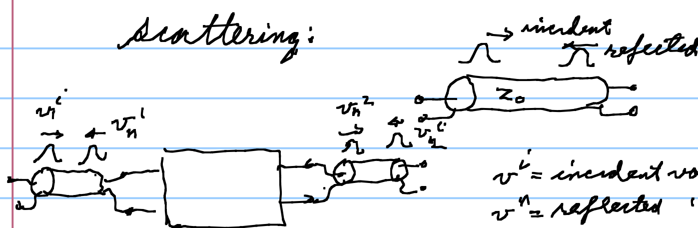
$= [O_{l,t} \ O_l] i_b$

$A_c v_b = B_c i_b$ $\quad c$ = connecting

$A v = B i \Rightarrow$ if these are port variables then a Y enters as $A = Y$, $B = 1_{\text{nxn}}$

a Z " " $A = 1_{\text{nxn}}$, $B = Z$

Scattering:



$e = 2v^i = v + Z_0 i$ $\qquad v^r = S v^i$,

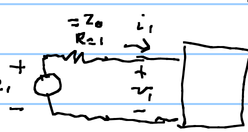
$2v^r = v - Z_0 i$

v^i = incident voltage $\qquad$ choose $Z_0 = 1$ by normalizing

v^r = reflected " "

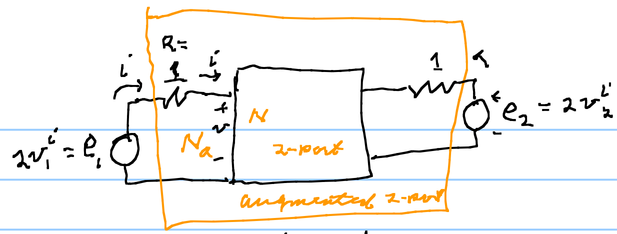
S = scattering matrix

$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = v^i, \quad v^r = \begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix}$



$v = e - Z_0 i$

$e + Ri = 2v^r$



$2v_1^i = e_1$ $\qquad$ $e_2 = 2v_2^i$

$Y_a e = i$

$$e = v + i = 2v^i$$
$$i = Y_a e$$
$$v = e - i$$
$$= e - Y_a e$$
$$= (1_2 - Y_a) e$$

$$2v^n = v - i$$
$$= (e - i) - Y_a e$$
$$= e - Y_a e - Y_a e$$
$$= (1_2 - 2Y_a) e$$

$S \Rightarrow v^n = S v^i$

$$Y_a v = Y_a (1_2 - Y_a) e = (1_2 - Y_a) Y_a e = (1_2 - Y_a) i$$

$$\Rightarrow Y_a v = (1_2 - Y_a) i \Rightarrow A = Y_a,\ B = (1_2 - Y_a) \text{ for } N_a$$

But $\left.\begin{array}{l} v = v^i + v^n \\ i = v^i - v^n \end{array}\right\}$ $A(v^i + v^n) = B(v^i - v^n)$

$$\Rightarrow (A+B) v^n = (B-A) v^i$$

$$\Rightarrow S = (B+A)^{-1}(B-A) = (1_2)^{-1}(1_2 - 2Y_a)$$